EXERCISE – II HINTS & SOLUTIONS

Sol.1 **A,B,C**

Its not asking homogeneous diff. equation

It is asking homogeneous function.

So (A) ✓ (B)✓ (C) ✓(D) ×

Sol.2 **C,D**

$y^2 + 4y'y + (y')^2 = 0$

$$y' = \frac{-4y \pm \sqrt{16y^2 - 4y^2}}{2}$$

$$\frac{dy}{dx} = \frac{-4y \pm 2\sqrt{3}y}{2}$$

+ve $\frac{dy}{y} = (-2 + \sqrt{3})\, dx$

$\ell ny = (-2 + \sqrt{3})\, x + \ell nC$

$y = k\, e^{(-2+\sqrt{3})x}$

–ve $y = k\, e^{(-2-\sqrt{3})x}$

Sol.3 **A,B**

$$\frac{dy}{dx} = \frac{-(x-y) \pm \sqrt{(x-y)^2 + 4xy}}{2y}$$

+ve $\frac{dy}{dx} = \frac{-(x-y)+(x+y)}{2y}$

$\frac{dy}{dx} = 1 \Rightarrow y = x + C$

passes through (3, 4)

$y - x = 1$

–ve $\frac{dy}{dx} = \frac{-(x-y)-(x+y)}{2y}$

$\frac{dy}{dx} = \frac{-x}{y}$

$\Rightarrow$ $ydy + xdx = 0$

$y^2 + x^2 = 25$

Sol.4 **A,B,D**

$\frac{dy}{dx} + y \cos x = \cos x$

I.F. $= e^{\int \cos x\, dx} = e^{\sin x}$

$y\, e^{\sin x} = \int e^{\sin x} \cos x\, dx = e^{\sin x} + C$

$y = 1 + ce^{-\sin x}$; $C = 0$ as (0, 1)

$y = 1$

Sol.5 **C**

$$\frac{dy}{dt} = k\sqrt{y} \Rightarrow \int_0^y \frac{dy}{\sqrt{y}} = \int_0^t k\, dt$$

$$2\sqrt{y} = kt \Rightarrow t = \frac{2\sqrt{y}}{k}$$

$t = 2 \times 2 \times 15 = 60$ min

Sol.6 **B**

$$\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$$

$\left.\begin{matrix} y = 0 \\ x = 0 \end{matrix}\right\rangle$ Two lines will satisfy above equation.

Sol.7 **A**

$$x^2 \frac{dy}{dx} \cdot \cos\frac{1}{x} - y \sin\frac{1}{x} = -1$$

$$\frac{dy}{dx} - \frac{y}{x^2} \tan\frac{1}{x} = \frac{-1}{x^2} \sec\frac{1}{x}$$

$$\text{I.F.} = e^{-\int \frac{1}{x^2} \tan\frac{1}{x} dx} = \sec\frac{1}{x}$$

$$y\left(\sec\frac{1}{x}\right) = \int \frac{-1}{x^2} \sec^2\frac{1}{x}\, dx$$

$$y\left(\sec\frac{1}{x}\right) = \tan\frac{1}{x} + C$$

$C = -1$

$$y\left(\sec\frac{1}{x}\right) = \tan\frac{1}{x} - 1$$

$$y = \sin\frac{1}{x} - \cos\frac{1}{x}$$

Sol.8 D

$$y = \frac{x}{\ell n(cx)}$$

$$y' = \frac{\ell n(x).1 - \frac{x}{cx}}{(\ell ncx)^2} = \frac{(\ell ncx) - \frac{1}{c}}{(\ell ncx)^2}$$

$$\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$$

$$\frac{\ell ncx - \frac{1}{c}}{(\ell ncx)^2} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$$

$$\frac{\frac{x}{y} - \frac{1}{c}}{\left(\frac{x}{y}\right)^2} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$$

$$\frac{y}{x} - \frac{1}{c\left(\frac{x}{y}\right)^2} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$$

$$\phi\left(\frac{x}{y}\right) = -\frac{1}{c}\left(\frac{y^2}{x^2}\right)$$

Sol.9 A

$$\int_0^x t\,y(t)dt = x^2 + y(x)$$

$xy(x) = 2x + y'(x)$; $y(a) = -a^2$

$x(y-2) = y'$

$$xdx = \frac{y'}{y-2}$$

$$\frac{x^2}{2} = \ell n(y-2) + C$$

$$y - 2 = e^{\left(\frac{x^2}{2} - c\right)}$$

$$\Rightarrow c = \frac{a^2}{2} - \ell n(-a^2 - 2)$$

$$y = 2 - (a^2 + 2)\, e^{\frac{x^2 - a^2}{2}}$$

Sol.10 A

$$\int_0^1 f(tx)\,dt = n f(x)$$

let $tx = z \Rightarrow dt = dz/x$

$$\int_0^x f(z).\frac{dz}{x} = n f(x)$$

$$\int_0^x f(z)\,dz = nx\, f(x)$$ use leibnitz to differentiate

$f(x) = n f(x) + n x f'(x)$

$nx f'(x) = (1 - n) f(x)$

$$\frac{f'(x)}{f(x)} = \left(\frac{1-n}{n}\right)\frac{1}{n}$$

$$\ell n f(x) = \left(\frac{1-n}{n}\right)\ell n x + \ell n c$$

$$\Rightarrow f(x) = k \,.\, x^{\left(\frac{1-n}{n}\right)}$$

Sol.11 C,D

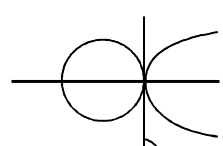
$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + \sin y + x^2 = 0$$

Order = 1, Degree = 1

Sol.12 A

$$Y - y = \frac{-dx}{dy}(X - x)$$

$$0 - y = \frac{-dx}{dy}(1 - x)$$



$ydy = (1 - x)\,dx$

$$\frac{y^2}{2} = x - \frac{x^2}{2} + C$$

passing through (0, 0)

$\Rightarrow C = 0$

$x^2 + y^2 - 2x = 0$

$y^2 = 4x$

$x = 0 \rightarrow$ is common target

Sol.13 A,D

$$\left(\frac{dy}{dx}\right)^2 - \left(\frac{dy}{dx}\right)(e^x + e^{-x}) + 1 = 0$$

$$\frac{dy}{dx} = \frac{(e^x + e^{-x}) \pm \sqrt{(e^x + e^{-x})^2 - 4}}{2}$$

$$\frac{dy}{dx} = \frac{(e^x + e^{-x}) \pm (e^x - e^{-x})}{2}$$

+ve $\frac{dy}{dx} = e^x \quad \Rightarrow y = e^x + C_1$

–ve $\frac{dy}{dx} = e^{-x} \quad \Rightarrow y = -e^{-x} + C_2$

Sol.14 A,C,D

$y = (A + Bx)\, e^{3x}$

$y' = 3(A + Bx)\, e^{3x} + Be^{3x} = e^{3x}\,(3A + B + 3Bx)$

$y' = 3y + Be^{3x}$

$y'' = 3y' + 3Be^{3x}$

By adding something and subtracting it will convert into

$y'' - 3y' = 3y + 3(B + 3A + 3Bx)\, e^{3x} - 12\,(A + Bx)\, e^{3x}$

$y'' - 3y' = 3y + 3y' - 12\,y$

$y'' - 6y' + 9y = 0$

$m = -6,\ n = 9$

Sol.15 C

$$2xy\,\frac{dy}{dx} = (x^2 + y^2 + 1)$$

Put $x^2 + y^2 + 1 = t$

$$2x + 2y\,\frac{dy}{dx} = \frac{dt}{dx}$$

$$2x^2 + 2xy\,\frac{dy}{dx} = x\,\frac{dt}{dx}$$

$$x\,\frac{dt}{dx} - 2x^2 = t \quad \Rightarrow \quad x\,\frac{dt}{dx} = t + 2x^2$$

$$\frac{dt}{dx} - \frac{t}{x} = 2x$$

$$\frac{t}{x} = 2x + C \Rightarrow \ t = 2x^2 + Cx$$

$x^2 + y^2 + 1 = 2x^2 + Cx$

$x^2 - y^2 + Cx - 1\ x = 0$

Sol.16 D

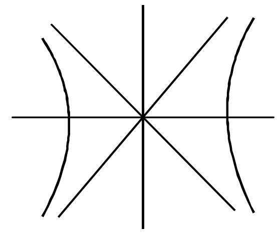
$f''(x) + f'(x) + f^2(x) = x^2$

put $f'(x) = 0$

$$\frac{d^2y}{dx} = x^2 - y^2 \le 0$$

$y^2 \ge x^2$

$|\,y\,| \ge |\,x\,|$



Sol.17 A

$x^2\,dy - y^2dx + x^2y^2\,dy - xy^3\,dy = 0$

$$\frac{1}{y^2}\,dy - \frac{1}{x^2}\,dx + dy - \frac{y}{x}\,dy = 0$$

$$d\left(\frac{1}{x} - \frac{1}{y}\right) = \left(\frac{y - x}{xy}\right) y\,dy$$

$$\frac{d\left(\frac{1}{x} - \frac{1}{y}\right)}{\left(\frac{1}{x} - \frac{1}{y}\right)} = ydy$$

$$\Rightarrow \ \ell n\left(\frac{1}{x} - \frac{1}{y}\right) = \frac{y^2}{2} + c$$

$$\ell n\left|\frac{y - x}{yx}\right| = \frac{y^2}{2} + c$$

$$\ell n\left|\frac{y - x}{yx}\right| = \frac{y^2}{2} + c$$

Sol.18 A,B,C,D

$$\left(\frac{dy}{dx}\right)^2 = \frac{y}{x} \quad \Rightarrow \quad \frac{dy}{dx} = \pm\,\frac{2}{\sqrt{x}}$$

Orthogonal Trajectory

$$\frac{dy}{dx} = \mp\,\frac{\sqrt{x}}{2}$$

By integrating

$$y + C = \pm\,\frac{x^{3/2}}{3} \quad \text{or} \quad 9(y + C)^2 = x^3$$